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# TIMATION Navigation Satellite System Constellation Study

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**NAVAL RESEARCH LABORATORY**  
Washington, D.C.

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## ABSTRACT

The Timation (Time Navigation) technique of passive ranging is a concept that could be expanded to provide a worldwide navigation service. Passive ranging is accomplished by measuring the time difference between electronic clocks located within the satellite and in the navigator's receiver. The Timation technique uses a highly stable, synchronized clock in the satellite; however, the navigator's clock stability may vary considerably, depending on the navigator's equipment and vehicle dynamics.

One-hundred and seven satellite constellations are discussed in terms of their influence on earth coverage and navigation fix accuracy. The recommended configuration derived is comprised of three planes with nine satellites ( $3 \times 9$ ) each in 8-hour circular orbits at a  $53^\circ$  inclination.

## PROBLEM STATUS

This is an interim report; work continues on the problem.

## AUTHORIZATION

NRL Problem R04-16  
Project A5385382/652C/1W34110000

Manuscript submitted January 13, 1972.

## TIMATION NAVIGATION SATELLITE SYSTEM CONSTELLATION STUDY

### INTRODUCTION

The Timation technique of passive ranging could be employed in a worldwide navigation service capable of providing instantaneous and continuous position fixes with high accuracy. The Timation Project is conducted by NRL under the sponsorship of the Naval Air Systems Command and the Navy Space Project Office (PM-16). The results of earlier analyses (1) indicated that a midaltitude constellation of satellites would be optimum for such a navigation service. The present work addresses the selection of suitable satellite constellation configurations that satisfy worldwide coverage and navigation fix accuracy requirements. The performance factor used here is based on a deterministic navigation fix using simultaneous passive range measurements, without assuming *a priori* range measurements or other external systems to aid in the fix determination. Passive ranging is accomplished by measuring the time difference between electronic clocks located within the satellite and in the navigator's receiver. The Timation technique uses a highly stable, synchronized clock in the satellite; however, the navigator's clock stability may vary considerably, depending on the navigator's equipment and vehicle dynamics. The deterministic navigation technique used is a direct extension of the range navigation technique described in Ref. 2.

### SATELLITE CONSTELLATION CONFIGURATIONS

A constellation of satellites may be configured in many ways; however, the practical problems associated with the deployment and operation of the satellites indicate a desirable range of constellation parameters. Reference 1 discusses a method for determining optimum altitudes for navigational satellites employing the Timation passive ranging concept, but the optimum constellation configuration was not determined from that study. A highly flexible computer program has been written to evaluate the merit of the satellite constellations considered. An extensive parameter set is employed to examine factors such as altitude, total number of satellites, launch injection parameters, and the relationships among the satellites in the constellation. The program is also capable of simulating any satellite in the constellation as being inoperative. This inoperative satellite mode is used to test the constellations for a degraded mode of system operation.

Figure 1 depicts a plane of nine satellites with circular orbits at an altitude of approximately 7500 naut mi (8-hr orbit). Equidistant satellite-to-satellite spacing within the plane provides uniform coverage. Figure 2 depicts a  $3 \times 9$  constellation (3 planes, 9 satellites per plane) inclined at  $53^\circ$  to the equator with a plane-to-plane spacing of  $120^\circ$ . Figure 3 gives a "snapshot" of the coverage for a  $3 \times 9$  constellation with an 8-hr period at a  $53^\circ$  inclination. With this  $3 \times 9$  constellation the navigator has available for navigation an average of eight satellites at any time (Fig. 4). The many other constellation configurations considered are summarized in Tables 1 and 2.

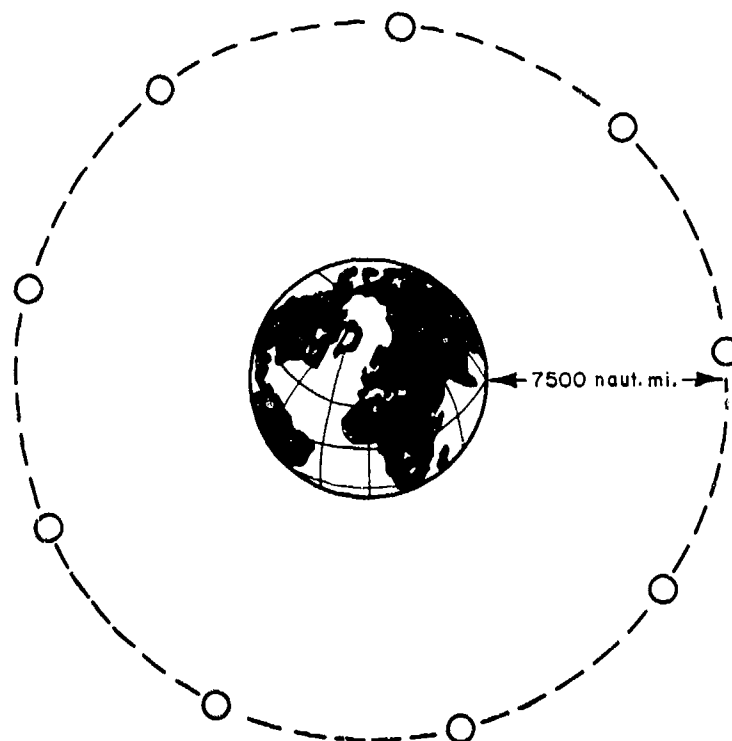


Fig. 1 — Plane of nine satellites with circular orbits at an altitude of approximately 7500 naut mi (8-hr orbit). Equidistant satellite-to-satellite spacing within the plane is used to provide uniform coverage.

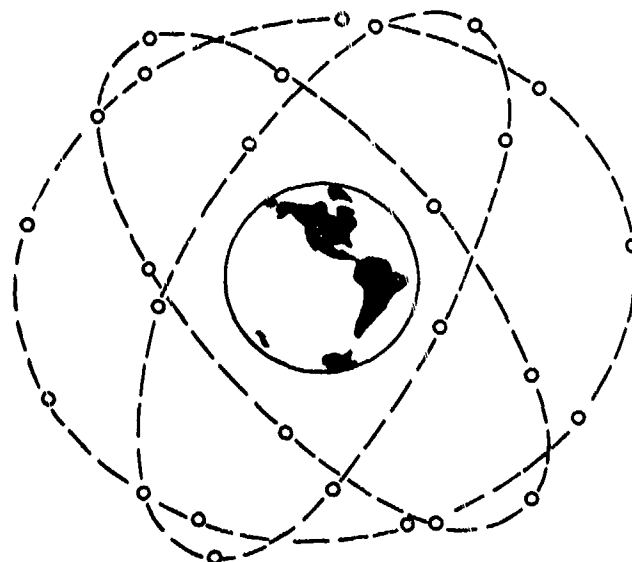


Fig. 2 —  $3 \times 9$  constellation (3 planes, 9 satellites per plane) inclined at  $53^\circ$  to the equator with a plane-to-plane spacing of  $120^\circ$

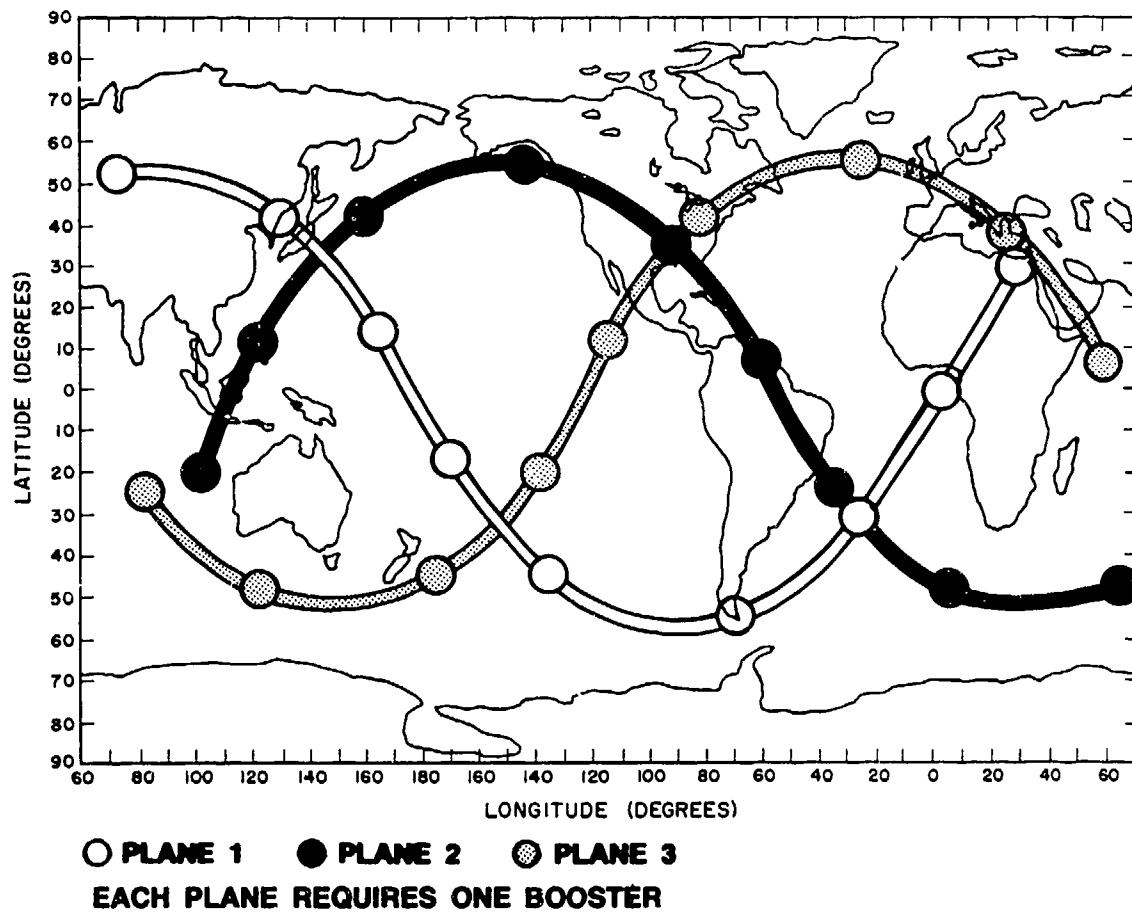


Fig. 3 -- Coverage for a  $3 \times 9$  constellation with an 8-hr period at a  $53^\circ$  inclination. The lead satellites in each plane are staggered with initial argument of perigee equal to  $40/3$ ,  $0$ ,  $-40/3$  degrees, respectively.

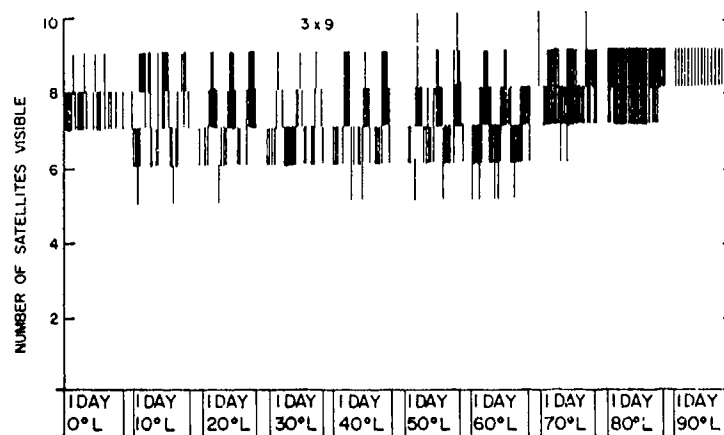


Fig. 4 -- Number of satellites in view as a function of time for a  $3 \times 9$  constellation inclined at  $53^\circ$  with a period of 8 hours and a  $10^\circ$  mask

Table 1  
Satellite Constellation Configurations

Number of Planes	Period (hr)	Inclination (degrees)	User Elevation Mask (degrees)	Number of Satellites per Plane											
				4	5	6	7	8	9	10	1	12			
3	4	90	5						x						
3	5	90	5						x						
3	6	53	5											x	
3	6	53	9											x	
3	6	53	10											x	
3	6	90	5						x						
3	8	40	5						x						
3	8	45	5						x						
3	8	50	5			x	x	x	x	x					
3	8	53	5				x	x	x	x				x	
3	8	53	9							x					
3	8	53	10				x	x	x			x		x	
3	8	53	15											x	
3	8	53	20											x	
3	8	55	4						x						
3	8	55	5	x	x				x						
3	8	55	10						x						
3	8	60	3						x						
3	8	60	4			x			x						
3	8	60	5			x	x	x	x	x					
3	8	70	4				x	x	x						
3	8	70	5			x	x	x							
3	8	80	4			x	x	x							
3	8	80	5			x	x	x							
3	8	90	0						x						
3	8	90	3					x							
3	8	90	4				x	x	x						
3	8	90	5			x	x	x	x	x				x	
3	8	90	10			x	x	x	x	x					
3	12	40	5						x						
3	12	45	5						x						
3	12	53	5						x	x					
3	12	53	10						x	x					
3	12	53	13						x						
3	12	53	14						x						
3	12	53	15						x						
3	12	55	5	x	x	x									
3	12	90	4					x							
3	12	90	5						x						
3	24	53	5				x	x	x						
3	24	53	10				x	x	x						
3	24	90	5				x								
3	24	90	10				x								
4	8	90	5		x										
4	8	90	5			x									
4	8	90	5				x								
4	8	50	5						x						
4	8	50	10						x						
4	8	50	15						x						
4	8	50	20						x						
2	8	55	5											x	
2	8	55	10											x	
2	12	55	5											x	

Legend: x indicates computer analysis of constellation.



Table 2  
Multi-Inclination Satellite Constellation Configurations

Number of Planes	Period (hrs)	Inclination (degrees)	User Elevation Mask (degrees)	Number of Satellites Per Plane		
				5	6	7
3	8	90	5		x	
1	8	0	5		x	
3	8	90	5			x
1	8	0	5			x
4	8	90	5	x		
1	8	0	5	x		

Legend: x indicates computer analysis of constellation.

The availability of synchronized clocks in each satellite enables the navigator to determine position instantaneously. This instantaneous position capability results from configuring the satellite constellation to allow viewing of at least four satellites at any instant of time. With four independent measurements available the navigator can determine his clock difference and solve for his position in three dimensions. The geometrical configuration of the satellites available for navigation depends on the particular constellation configuration and the navigator's position. Hence, it is desirable to compute a performance factor which can be used to isolate the geometrical quality of a given constellation. With the use of a suitable performance factor the impact of constellation configuration on navigational accuracy can be examined in detail.

#### GDOP PERFORMANCE FACTOR

The measure of performance that will be used is geometric dilution of precision (GDOP), which is defined as the square root of the sum of variances of the coordinates of interest. Rather than use terms such as 3D-GDOP, or 2D-GDOP, special terms will be defined for each GDOP of interest. The term position dilution of precision (PDOP) is defined as the three-dimensional GDOP, which is the square root of the sum of the navigator's x, y, and z coordinate variances. PDOP is independent of the particular xyz frame of reference used by the navigator; furthermore, it is equal to the PDOP value obtained using spherical coordinates, i.e., latitude, longitude, and altitude. The term CEP is defined as the two-dimensional GDOP, where the xy plane coincides with the navigator's horizontal plane. The altitude GDOP is defined as the square root of the variance for the navigator's local vertical coordinate. The term GDOP itself will be used to denote the square root of the sum of the xyz position variances and the clock difference variance.

The range measurements are assumed to be independent with unit variance, so the value of GDOP will appear as a pure number. The RMS navigation error is then given by the product of the GDOP factor and the total range error budget (which may be expressed in nautical miles or meters).

The GDOP values obtained for the position coordinates will depend on the ranges, the mathematical model, the total number of estimated parameters, and the observability

of the desired parameters. The model used solves for four parameters; three parameters are the navigator's x, y, and z coordinates, and the fourth parameter is the clock difference between the navigator's clock and any of the satellite clocks. The four-parameter solution will give a measure of the instantaneous position-fixing capability.

The solution does not incorporate any previous history of range measurements into the estimate of the navigator's current position. This total disregard of past-measurement history has the interesting feature of yielding a position solution that is virtually independent of the stability of the user's clock. However, the previous position estimate may be used in the current position estimate without assuming that the user has a highly stable clock. The observability of the navigator's position is measured by the size of the PDOP factor.

Another type of solution of interest will be used where it is assumed that the user clock has been updated. In this case the clock difference is known, and the navigator's xyz position coordinates are the only quantities to be estimated. This type of solution will be referred to as the good-user clock solution.

To obtain a solution for the navigator's position of either type (clock known or clock unknown), the navigator must have either three or four (respectively) satellites in view at any instant of time from any position on or near the earth. Reference 3 discusses systematic methods for determining continuously visible satellite constellations, however, for the satellite constellations considered in this report, the number of satellites in view was obtained by direct computation of the number visible from any point in the worldwide mesh. The direct computation technique also allows the inclusion of a minimum elevation mask, which disregards all satellites below a specified elevation angle.

## ANALYSIS OF PDOP PRESENTATION

The results of various simulated satellite configurations will be presented graphically in two basic forms. The first type of presentation is a PDOP-versus-time plot using a compressed scale for the time axis. The time increment has been chosen to be 10 minutes for all cases shown. For a particular configuration the constellation is started at time  $t = 0$  and run for 1 day, computing the PDOP for a navigator stationed at selected longitudes with latitude increments of  $10^\circ$ , starting at the equator and stopping at the North Pole. The constellations are symmetric with respect to the equator so it is not necessary to perform the PDOP computations for the Southern Hemisphere latitudes. The total number of PDOP computations performed for each of the PDOP graphs is 1440. Figures 5 through 31 use this type of format.

## 3 × 7 CONSTELLATION

The 3 × 7 constellation was chosen as a starting point in the GDOP discussion because it readily demonstrates several important factors involved in this study. The 3 × 7 constellation uses uniformly spaced satellites every 51.4 degrees with a plane-to-plane spacing of 120 degrees. Figure 5 gives a plot of PDOP for a 3 × 7 constellation inclined at  $90^\circ$  with a period of 8 hours.\* The mode of navigation used for the PDOP values computed assumes

\*The PDOP values on Fig. 5 use a three-parameter xyz solution; all other GDOP graphs use a four-parameter (navigator's xyz plus clock) solution.

that the navigator has a good clock. The good-user clock assumption means that the navigator will solve for just his xyz coordinates and not for the clock bias parameter. A PDOP value of approximately 1.5 is uniformly maintained for all latitudes.

Figure 6 is the same constellation as Fig. 5; however, the four-parameter (xyz plus clock bias) solution is plotted. A comparison of Figs. 5 and 6 immediately demonstrates the value of a good-user clock assumption. Figure 6 shows that spikes (high PDOP values) are obtained near the equator and again at  $60^\circ$  to  $70^\circ$  latitudes. Note that at a  $90^\circ$  latitude no spikes occur and the average PDOP value is near 2.0. The results at a  $90^\circ$  latitude indicate that the PDOP values can be balanced by reducing the inclination of the satellite planes. Figure 7 demonstrates that the PDOP values can be balanced by using a  $53^\circ$  inclination for the constellation, and Fig. 8 demonstrates the effect of increasing the mask on the observations to  $10^\circ$  for a  $90^\circ$  inclination. A navigator located near rugged terrain might require a  $10^\circ$ , or higher, mask. A similar effect for a  $53^\circ$  inclination constellation is shown in Fig. 9; Fig. 10 demonstrates that increasing the period to 24 hours gives an improvement in PDOP values over Figs. 6 or 7. The increase of mask to  $10^\circ$  in Fig. 11 causes some spikes in the coverage.

Figure 12 indicates that for a 24-hr orbit no significant improvement results by using a  $53^\circ$  inclination instead of the  $90^\circ$  inclination in Fig. 10. An increase in spikes occurs as a result of using a  $10^\circ$  mask for a 24-hr orbit with a  $53^\circ$  inclination in Fig. 13. It is apparent from these results with a  $3 \times 7$  constellation that additional satellites are required to improve the coverage.

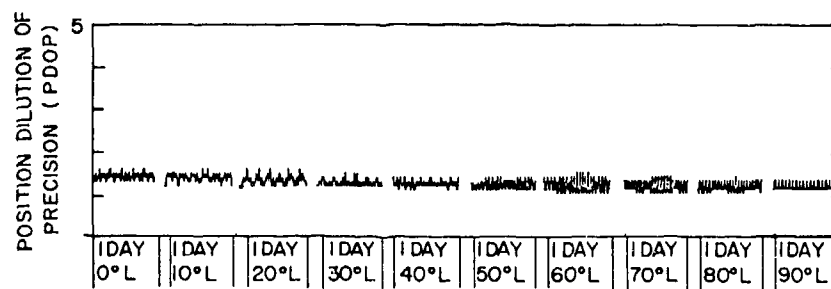


Fig. 5 — PDOP plot for a  $3 \times 7$  constellation inclined at  $90^\circ$  with a period of 8 hr and a  $5^\circ$  mask. A good-user clock assumption was made for this figure.

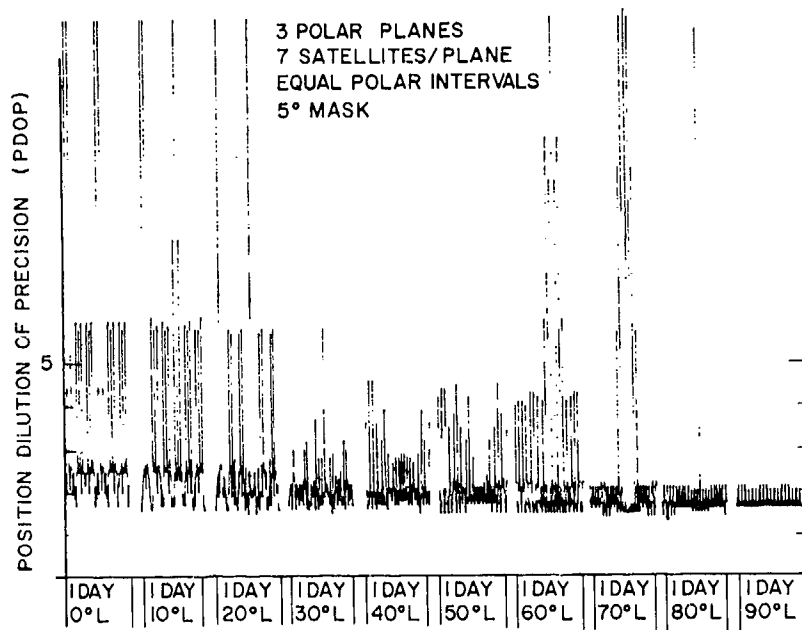


Fig. 6 — PDOP plot for a  $3 \times 7$  constellation inclined at  $90^\circ$  with a period of 8 hr and a  $5^\circ$  mask. A four-parameter solution is used instead of a three-parameter xyz solution.

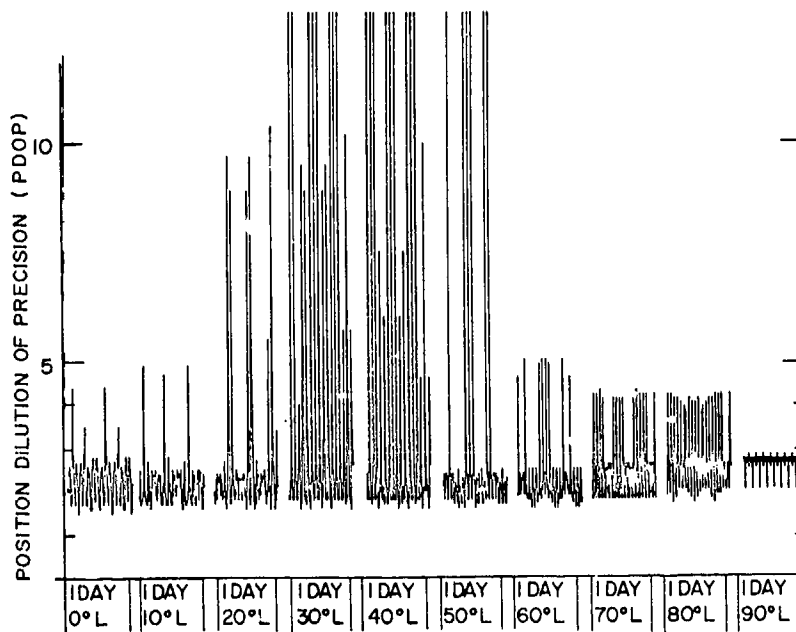


Fig. 7 — PDOP plot for a  $3 \times 7$  constellation inclined at  $53^\circ$  with a period of 8 hr and a  $5^\circ$  mask

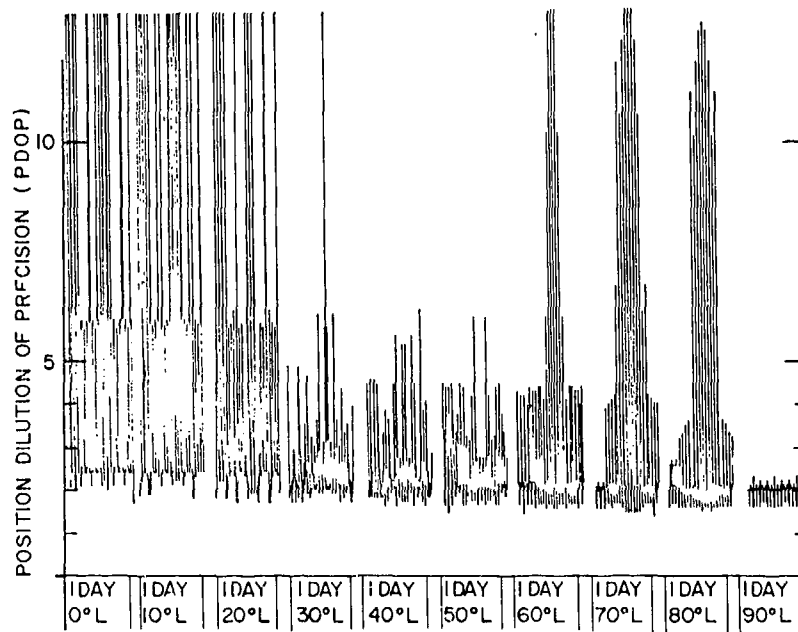


Fig. 8 — PDOP plot for a  $3 \times 7$  constellation inclined at  $9^\circ$  with a period of 8 hr and a  $10^\circ$  mask

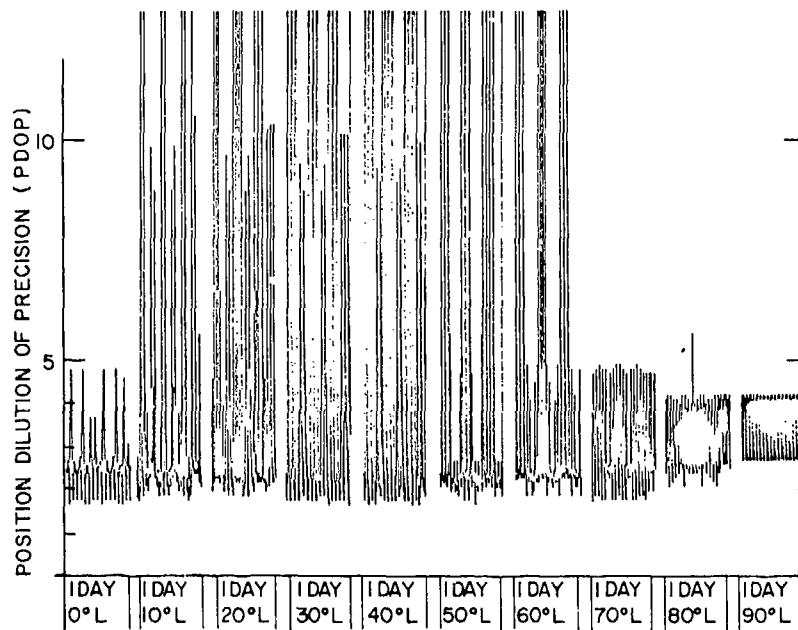


Fig. 9 — PDOP plot for a  $3 \times 7$  constellation inclined at  $53^\circ$  with a period of 8 hr and a  $10^\circ$  mask

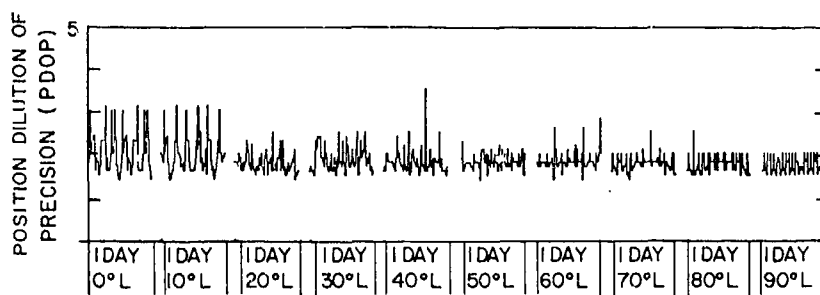


Fig. 10 — PDOP plot for a  $3 \times 7$  constellation inclined at  $90^\circ$  with a period of 24 hr and a  $5^\circ$  mask

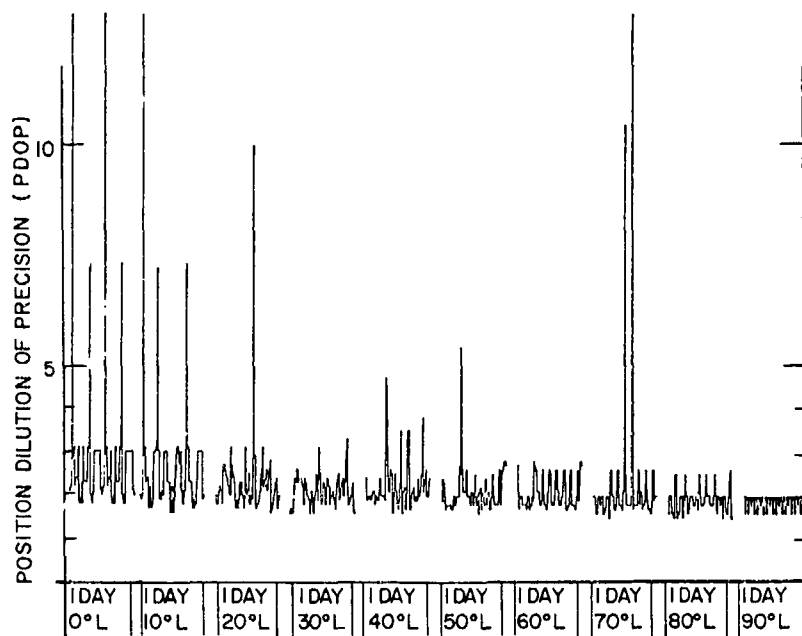


Fig. 11 — PDOP plot for a  $3 \times 7$  constellation inclined at  $90^\circ$  with a period of 24 hr and a  $10^\circ$  mask

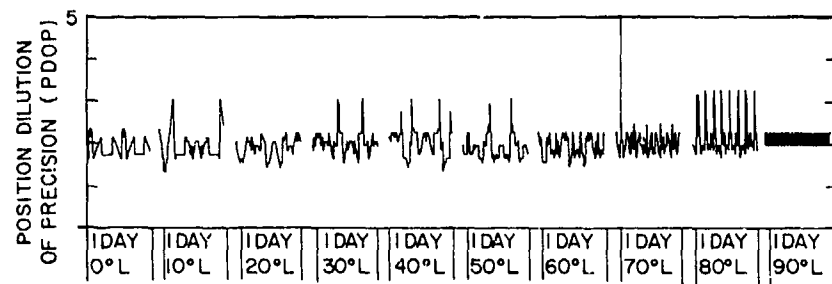


Fig. 12 — PDOP plot for a  $3 \times 7$  constellation inclined at  $53^\circ$   
with a period of 24 hr and a  $5^\circ$  mask

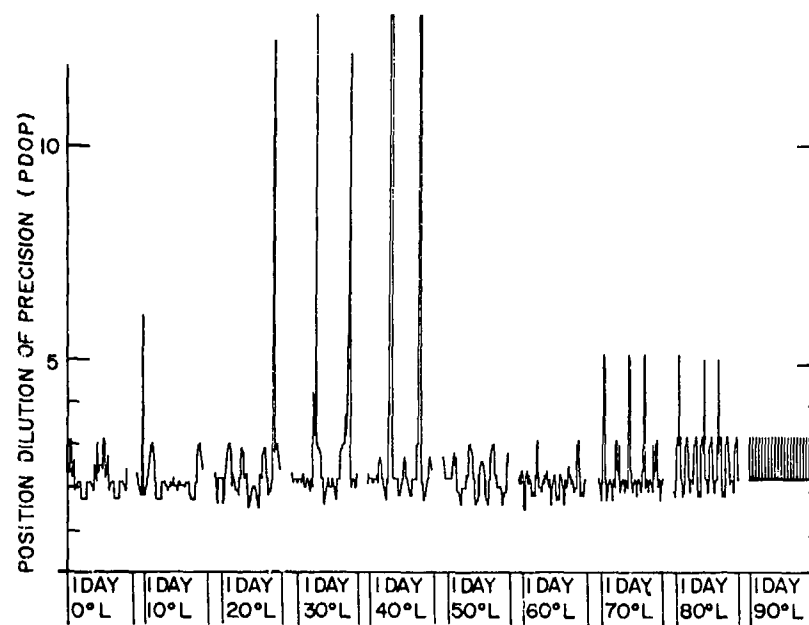


Fig. 13 — PDOP plot for a  $3 \times 7$  constellation inclined at  $53^\circ$   
with a period of 24 hr and a  $10^\circ$  mask

### 3 × 8 CONSTELLATION

The 3 × 8 constellation uses uniformly spaced satellites every 45 degrees with a plane-to-plane spacing of 120 degrees.

An improvement over Fig. 6 in PDOP coverage is shown in Fig. 14. This improvement results from increasing the number of satellites to 24 in a 3 × 8 constellation with an 8-hr period. Only a few spikes occur near the equator in Fig. 14; however, a definite taper is evident as a result of the overcoverage near the poles. All but one spike disappears if the mask is lowered from 5° to 4° in Fig. 15, and this spike vanishes when lowering the mask further to 3.95°. If the mask is increased to 10°, coverage deteriorates at the equator (Fig. 16). Figure 17 shows an improvement over Fig. 14 as a result of lowering the inclination from 90° to 53° for an 8-hr period. Several spikes occur in the PDOP of Fig. 18 as a result of raising the mask to 10°. The 12-hr orbit in Fig. 19 has two small spikes at the 80° latitude; several more small spikes appear with an increase to a 10° mask in Fig. 20. Increasing the inclination to 90° and the period to 24 hr produced similar results for the 5° mask (Fig. 21) and the 10° mask (Fig. 22). A small improvement in PDOP coverage is evident for the 53° inclination in Fig. 23 as opposed to Fig. 21 with a 90° inclination constellation. Figure 24 shows the expected increase of PDOP values over Fig. 22 as a result of a 10° elevation mask. The 3 × 8 constellation shows a considerable improvement over the 3 × 7 constellation in PDOP coverage.

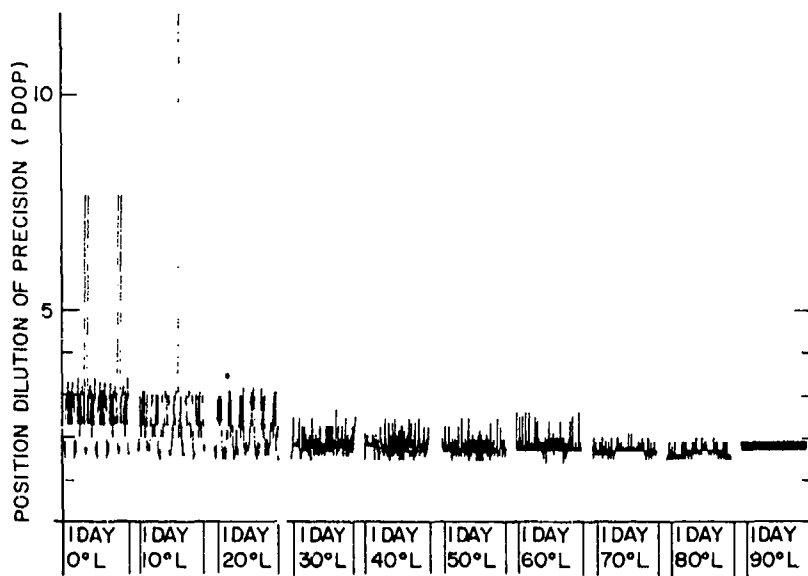


Fig. 14 — PDOP plot for a 3 × 8 constellation inclined at 90° with a period of 8 hr and a 5° mask



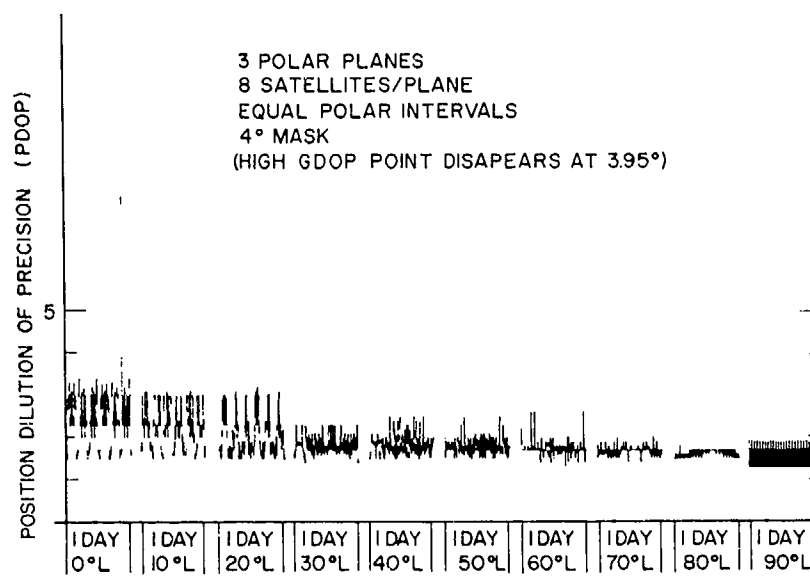


Fig. 15 — PDOP plot for a  $3 \times 8$  constellation inclined at  $90^\circ$   
with a period of 8 hr and a  $4^\circ$  mask

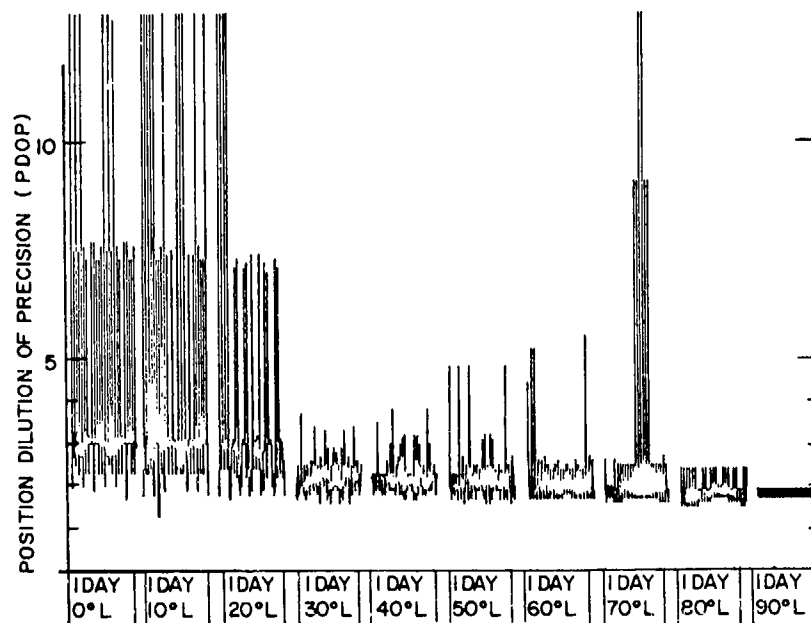


Fig. 16 — PDOP plot for a  $3 \times 8$  constellation inclined at  $90^\circ$   
with a period of 8 hr and a  $10^\circ$  mask

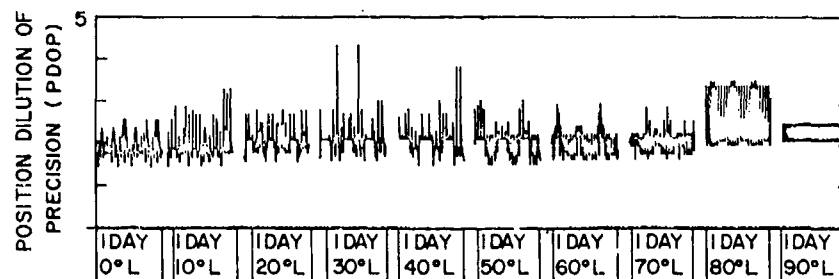


Fig. 17 — PDOP plot for a  $3 \times 8$  constellation inclined at  $53^\circ$  with a period of 8 hr and a  $5^\circ$  mask

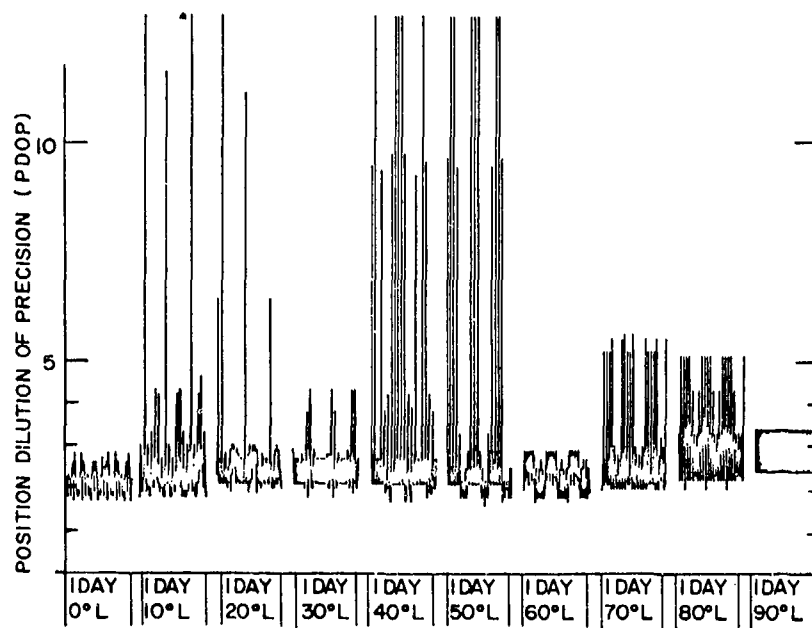


Fig. 18 — PDOP plot for a  $3 \times 8$  constellation inclined at  $53^\circ$  with a period of 8 hr and a  $10^\circ$  mask

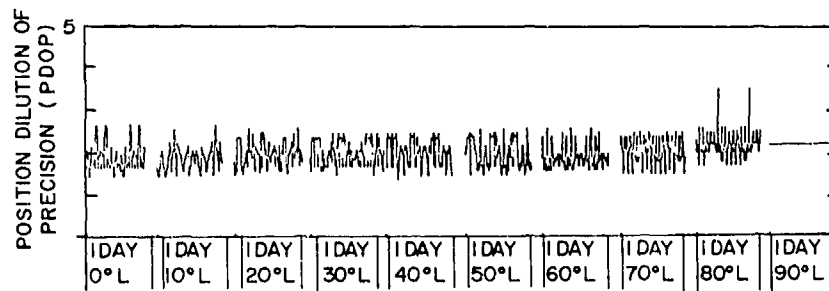


Fig. 19 — PDOP plot for a  $3 \times 8$  constellation inclined at  $53^\circ$  with a period of 12 hr and a  $5^\circ$  mask

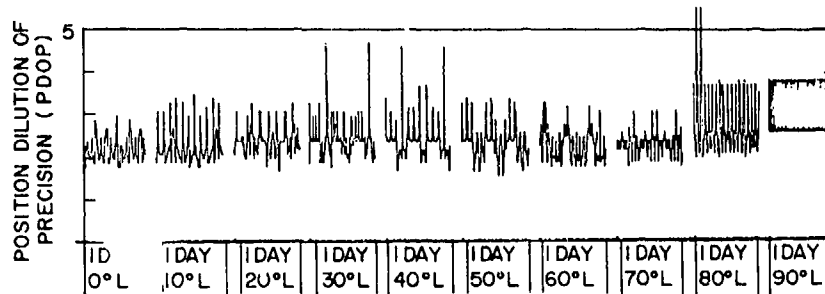


Fig. 20 — PDOP plot for a  $3 \times 8$  constellation inclined at  $53^\circ$  with a period of 12 hr and a  $10^\circ$  mask

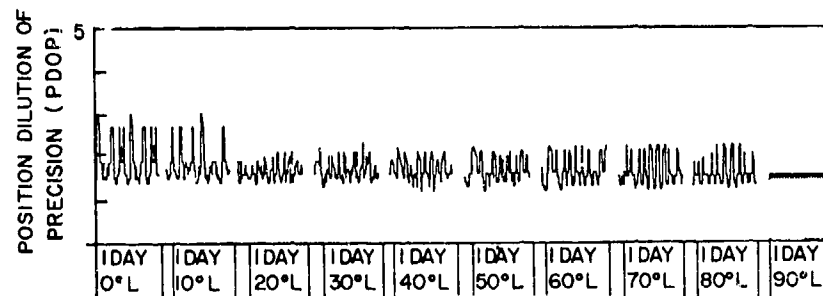


Fig. 21 — PDOP plot for a  $3 \times 8$  constellation inclined at  $90^\circ$  with a period of 24 hr and a  $5^\circ$  mask

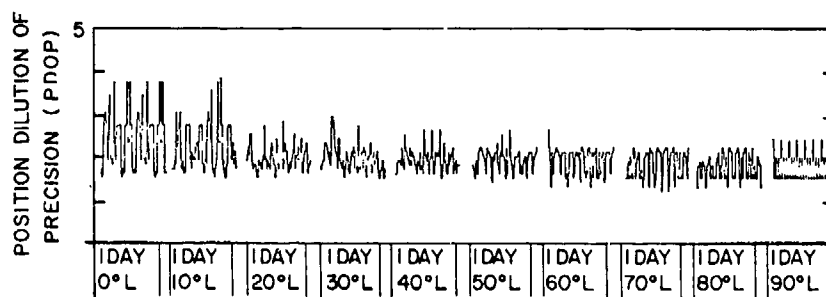


Fig. 22 — PDOP plot for a  $3 \times 8$  constellation inclined at  $90^\circ$  with a period of 24 hr and a  $10^\circ$  mask

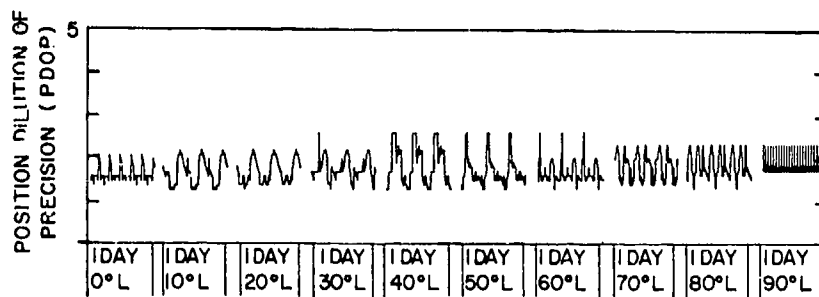


Fig. 23 — PDOP plot for a  $3 \times 8$  constellation inclined at  $53^\circ$  with a period of 24 hr and a  $5^\circ$  mask

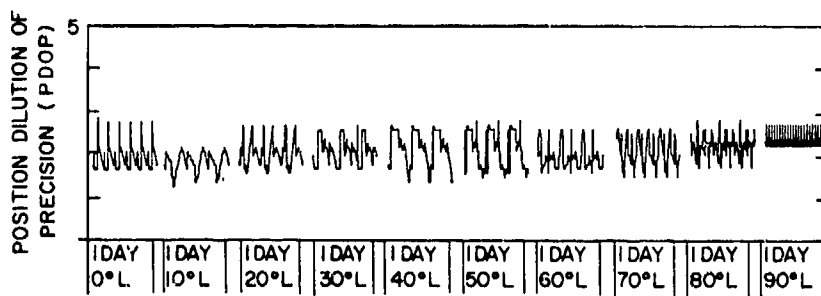


Fig. 24 — PDOP plot for a  $3 \times 8$  constellation inclined at  $53^\circ$  with a period of 24 hr and a  $10^\circ$  mask

### 3 × 9 CONSTELLATION

The 3 × 9 constellation uses uniformly spaced satellites every 40 degrees with a plane-to-plane spacing of 120 degrees.

The 3 × 9 constellation with an 8-hr period and a  $30^\circ$  inclination in Fig. 25 shows an improvement over the similar 3 × 8 configuration in Fig. 14. The effect of an increase of the mask to  $10^\circ$  is evident in Fig. 26. Figure 27 compares  $5^\circ$  and  $10^\circ$  user elevation masks for a 3 × 9 constellation with an 8-hr period and a  $53^\circ$  inclination. Figure 28 clearly demonstrates the balancing of PDOP coverage as a function of constellation inclination. Figure 29 compares  $10^\circ$  and  $15^\circ$  masks for a 12-hr period constellation at  $53^\circ$  inclination and shows that a few values of PDOP occur that are above 5.0 for a  $15^\circ$  mask.

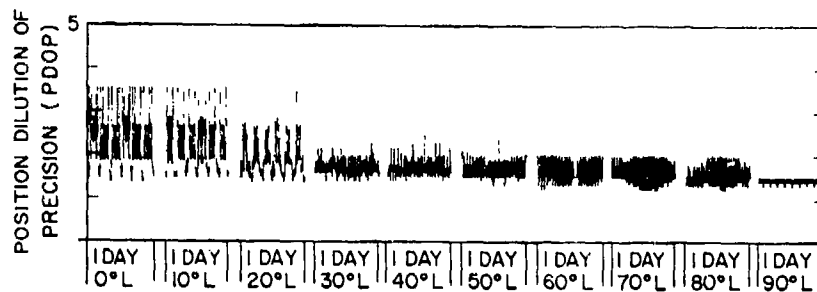


Fig. 25 — PDOP plot for a 3 × 9 constellation inclined at  $90^\circ$  with a period of 8 hr and a  $5^\circ$  mask

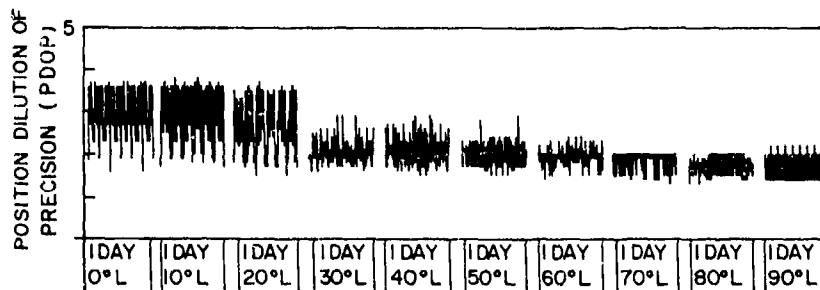


Fig. 26 — PDOP plot for a 3 × 9 constellation inclined at  $90^\circ$  with a period of 8 hr and a  $10^\circ$  mask

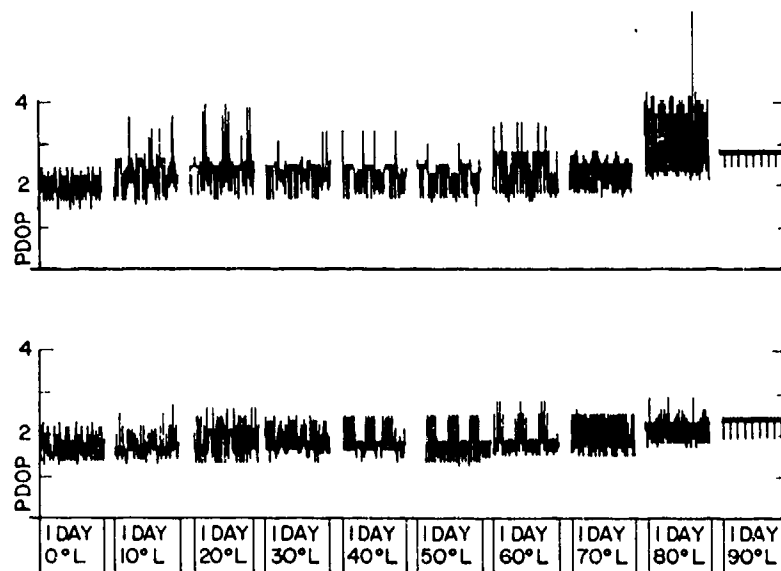


Fig. 27 — Comparison of 5° and 10° user elevation masks for a 3 x 9 constellation with 8-hr orbits at 53° inclinations

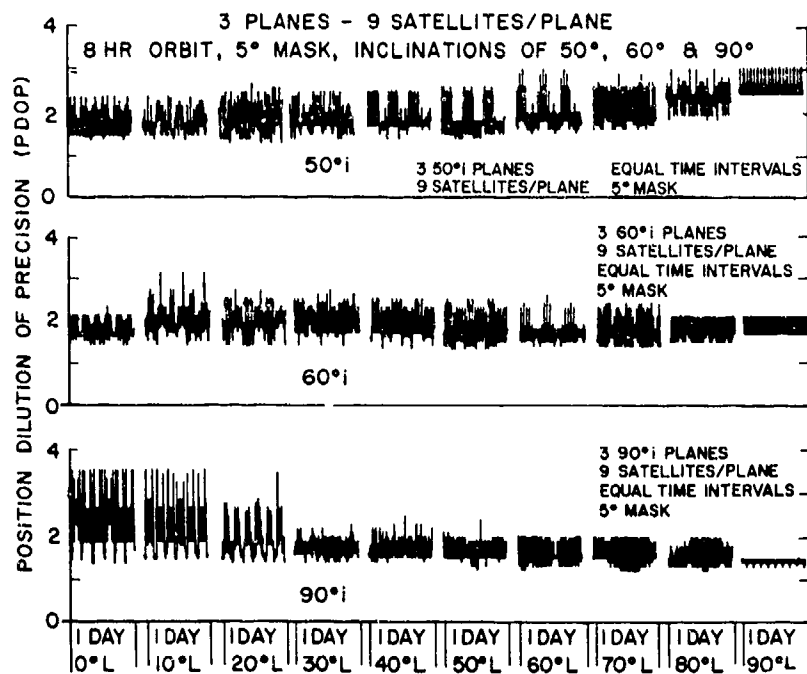


Fig. 28 — Comparison of 50°, 60°, and 90° inclinations for a 3 x 9 constellation with 8-hr orbits and 5° masks

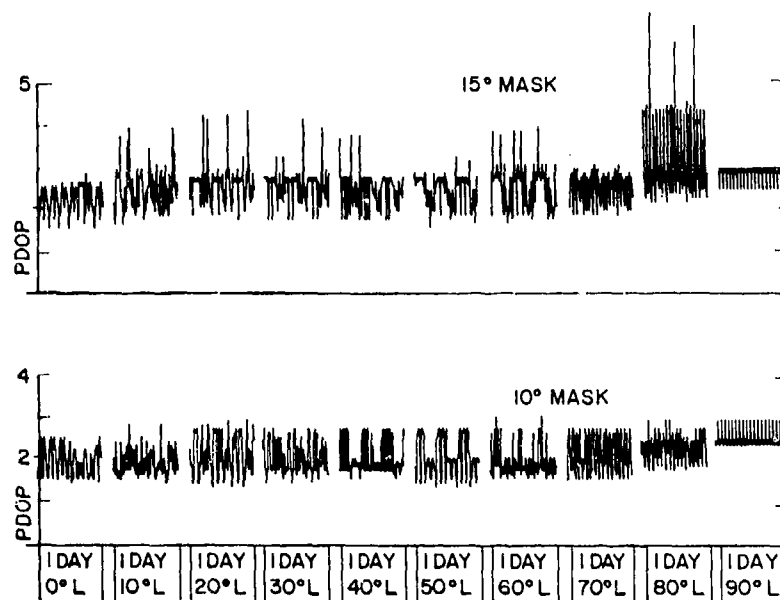


Fig. 29 — Comparison of 10° and 15° masks for a 3 × 9 constellation with 12-hr orbits at 53° inclinations

### 3 × 12 CONSTELLATION

The 3 × 12 constellation uses uniformly spaced satellites every 30 degrees with a plane-to-plane spacing of 120 degrees.

The 3 × 12 constellation was investigated to determine the improvement in PDOP coverage using this increased number of satellites. Figure 30 gives the PDOP coverage for a 53° inclination constellation with an 8-hr period and a 5° user elevation mask. Figures 31 through 33 are PDOP plots for the same constellation using different user elevation masks — 10°, 15°, and 20°, respectively. These results indicate that a higher elevation mask can be used for 36 satellites in the constellation.

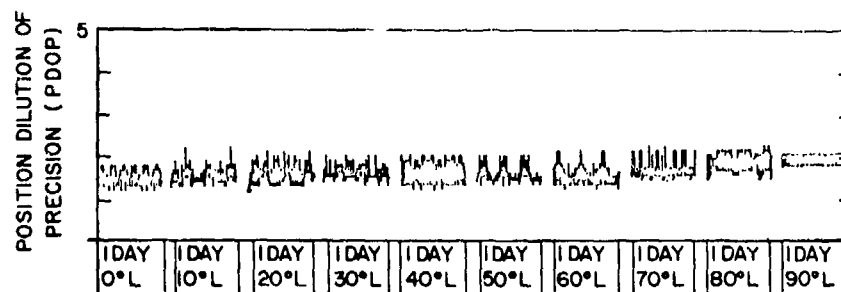


Fig. 30 — PDOP plot for a 3 × 12 constellation inclined at 53° with a period of 8 hr and a 5° mask

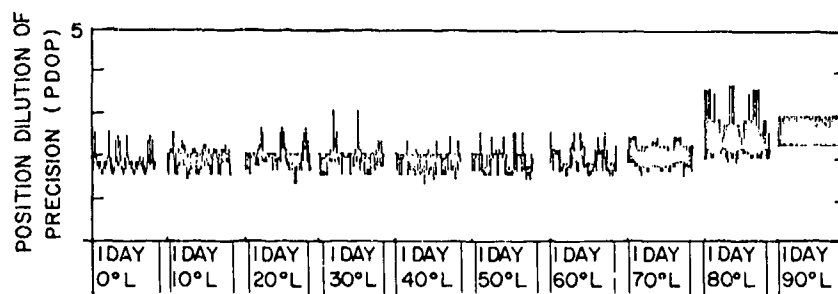


Fig. 31 — PDOP plot for a  $3 \times 12$  constellation inclined at  $53^\circ$  with a period of 8 hr and a  $10^\circ$  mask

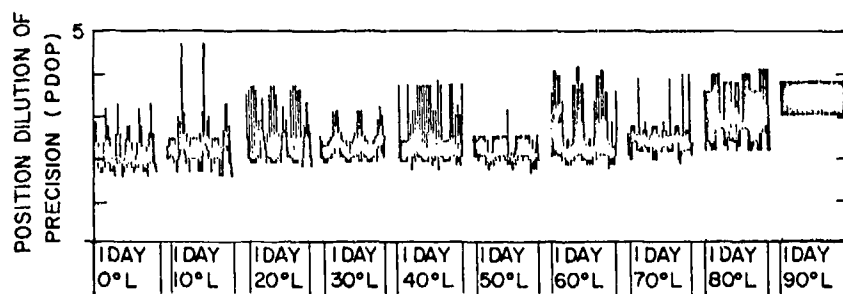


Fig. 32 — PDOP plot for a  $3 \times 12$  constellation inclined at  $53^\circ$  with a period of 8 hr and a  $15^\circ$  mask

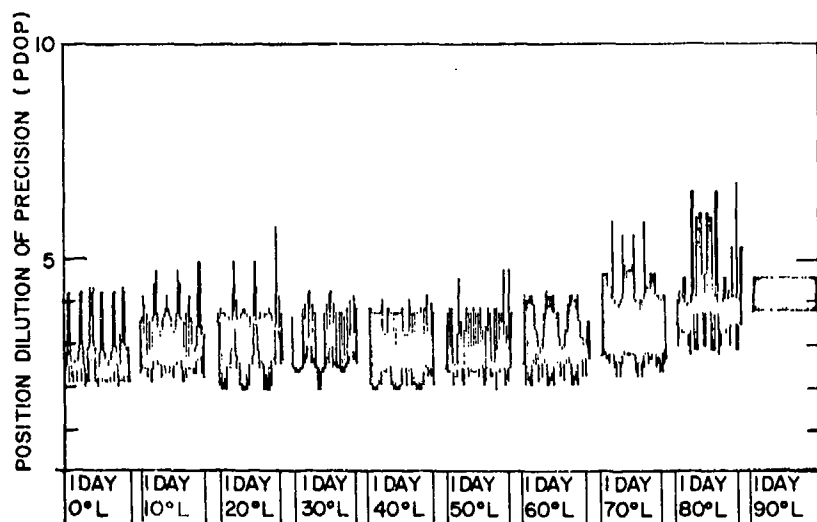


Fig. 33 — PDOP plot for a  $3 \times 12$  constellation inclined at  $53^\circ$  with a period of 8 hr and a  $20^\circ$  mask



### INOPERATIVE SATELLITE CONSTELLATION

The  $3 \times 9$  constellation was chosen to evaluate the effect on PDOP coverage resulting from one satellite in the constellation becoming inoperative via enemy attack or system malfunction. Table 3 summarizes these inoperative satellite constellation configurations. When the first satellite in the second plane becomes inoperative, a user at the equator experiences an increase in the PDOP values to slightly above 4, and users at higher latitudes experience little change in PDOP values (Fig. 34). When the second satellite in the second plane becomes inoperative, the PDOP values increase occasionally at all latitudes except at  $90^\circ$  (Fig. 35). A user with a  $10^\circ$  elevation mask will experience a more pronounced effect on PDOP values (Fig. 36). The results from other choices of one inoperative satellite in a  $3 \times 9$  constellation with an 8-hr period at a  $53^\circ$  inclination indicate that the PDOP values are slightly worse than for the  $3 \times 8$  constellation PDOP results given by Fig. 17. This result is explained by the fact that none of the remaining eight satellites in the plane with the inoperative satellite was repositioned.

Table 3  
Inoperative Satellite Constellation Configurations

Number of Planes	Period (hr)	Inclination (degrees)	User Elevation Mask (degrees)	Number of Satellites Per Plane					Number of Inoperative Satellites
				8	9	10	11	12	
3	8	53	5		x				1
3	8	53	10		x				1
3	8	55	5		x				1
3	8	90	5		x				1
3	12	53	5		x				1
3	12	90	5		x				1
3	8	90	4	x					1

Legend: x indicates computer analysis of constellation.

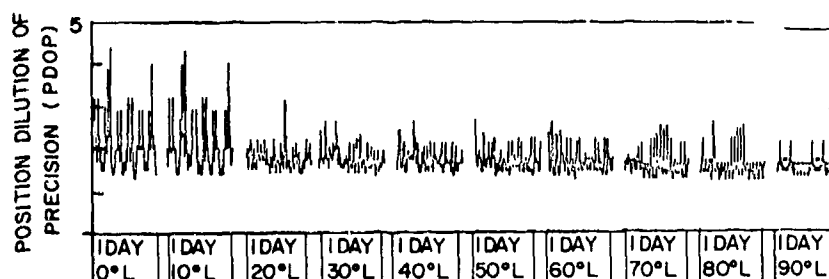


Fig. 34 — Inoperative satellite PDOP plot of a  $3 \times 9$  constellation with satellite (2,1) inoperative and a  $5^\circ$  mask

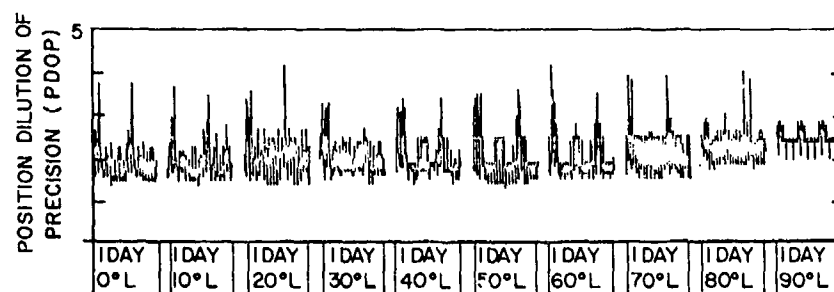


Fig. 35 — Inoperative satellite PDOP plot of a  $3 \times 9$  constellation with satellite (2,2) inoperative and a  $5^\circ$  mask

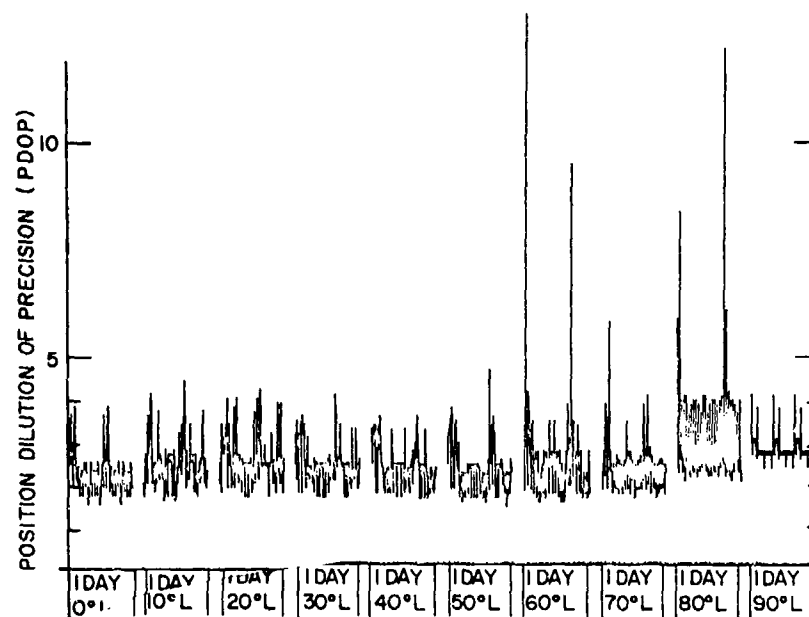


Fig. 36 — Inoperative satellite PDOP plot of a  $3 \times 9$  constellation with satellite (2,2) inoperative and a  $10^\circ$  mask

## ANALYSIS OF PERCENTAGE PRESENTATION

The second type of graphical presentation is used to summarize the percentage of PDOP values equal to or less than a given number for a one-day computer run of a constellation with the navigator stationed from  $0^\circ$  to  $90^\circ$  latitude ( $10^\circ$  increments) in the worldwide mesh. Figures 37 through 45 use this format. A logarithmic scale is used for the percentage axis, which emphasizes small differences near 100% coverage for a particular constellation. A vertical asymptote indicates 100% worldwide coverage with all PDOP values below the corresponding number on the PDOP axis, while a horizontal asymptote indicates that a few locations exist in the worldwide mesh where a navigator would experience a high PDOP value at some time during the day. Table 4 summarizes all the percentage graphs that are plotted.

Table 4  
Percent Coverage Graphs

Figure Number	Satellite Constellations	Inclination (degrees)	Period (hr)	User Elevation Mask (degrees)
37	$3 \times 9$	53	8	5
38	$3 \times 9$	53	8	5
	$3 \times 8$			
	$3 \times 7$			
39	$3 \times 9$	53	8	10
	$3 \times 8$			
	$3 \times 7$			
40	$3 \times 8$	53	24	5
	$3 \times 7$			
41	$3 \times 8$	53	24	10
	$3 \times 7$			
42	$3 \times 9$	90	8	5
	$3 \times 8$			
	$3 \times 7$			
43	$3 \times 9$	90	8	10
	$3 \times 8$			
	$3 \times 7$			
44	$3 \times 8$	90	24	5
	$3 \times 7$			
45	$3 \times 8$	90	24	10
	$3 \times 7$			

A  $3 \times 9$  constellation at a  $53^\circ$  inclination with an 8-hour period will give worldwide PDOP coverage with a maximum PDOP value of 3.0 for  $10^\circ$  latitude increments and 10-minute time increments (Fig. 37). The CEP has a maximum value of 1.5 and the altitude coordinate variance has a maximum value of 2.8, which indicates that the altitude will be the poorest determined coordinate in the three-dimensional fix. The navigator's clock has a maximum GDOP value of 1.8.

Figure 38 indicates that approximately 90% of the time a PDOP value of 2.8 would be available with either a  $3 \times 7$ ,  $3 \times 8$ , or  $3 \times 9$  constellation. For the remaining 10% of the time the  $3 \times 9$  constellation is superior to a  $3 \times 7$  or  $3 \times 8$  constellation. An increase of the elevation mask to  $10^\circ$  increases about 0.1% of the PDOP values for a  $3 \times 9$  constellation to greater than 4.0 (Fig. 39). For the same elevation mask ( $10^\circ$ ), a  $3 \times 7$  constellation would have PDOP spikes 10% of the time. The comparison of the  $3 \times 7$  and  $3 \times 8$  constellations with 24-hr periods in Fig. 40 shows that the  $3 \times 8$  PDOP coverage is approximately equal to a  $3 \times 9$  constellation with an 8-hr period. Figure 41 illustrates the effect of increasing the elevation mask to  $10^\circ$ .

Increasing the inclination to  $90^\circ$  for the three constellations in Fig. 42 gives higher PDOP values as compared to the  $53^\circ$  inclination in Fig. 38. Figure 43 shows the effect of an increase of elevation mask to  $10^\circ$ . For a 24-hr orbit the difference shown in Fig. 44 for PDOP coverage between the  $3 \times 7$  and  $3 \times 8$  constellations is not as significant as it is for the 8-hr orbit given in Fig. 42. The  $90^\circ$  inclination constellation in Fig. 44 may also be compared with the  $53^\circ$  inclination constellation in Fig. 40 to show that the PDOP coverage improvement of a  $3 \times 8$  constellation compared to a  $3 \times 7$  constellation is not as significant for a 24-hr orbit as it is for an 8-hr orbit. Figure 45 shows the effect of increasing the elevation mask to  $10^\circ$ .

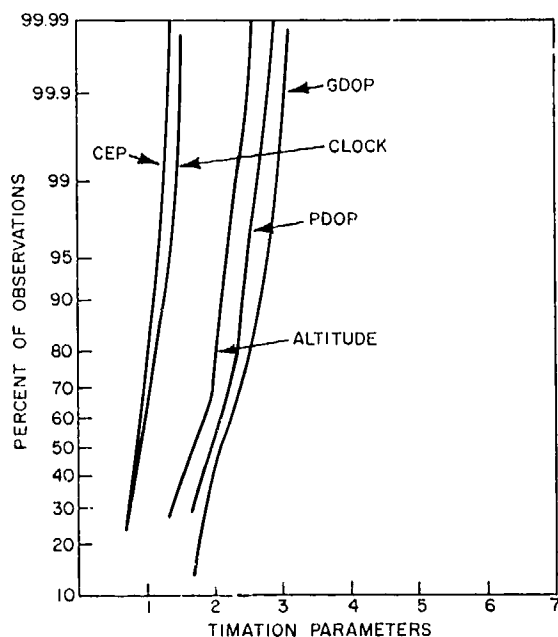


Fig. 37 — Percentile coverage versus Timation parameters of a  $3 \times 9$  constellation at  $53^\circ$  inclination with an 8-hr period and  $5^\circ$  mask

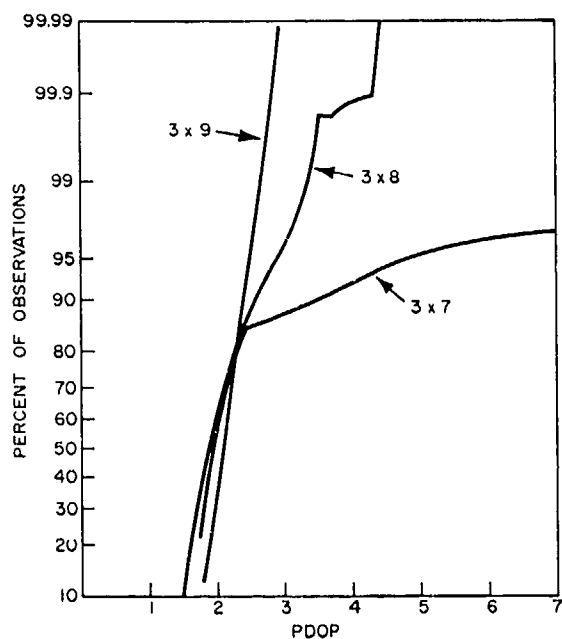


Fig. 38 — Percentile coverage versus PDOP value plot of  $3 \times 7$ ,  $3 \times 8$ , and  $3 \times 9$  constellations at  $53^\circ$  inclination with an 8-hr period and a  $5^\circ$  mask

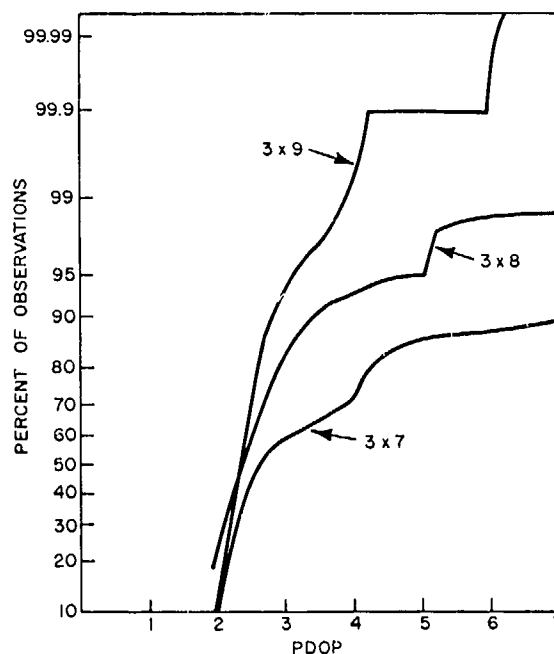


Fig. 39 — Percentile coverage versus PDOP value plot of  $3 \times 7$ ,  $3 \times 8$ , and  $3 \times 9$  constellations at  $53^\circ$  inclination with an 8-hr period and a  $10^\circ$  mask

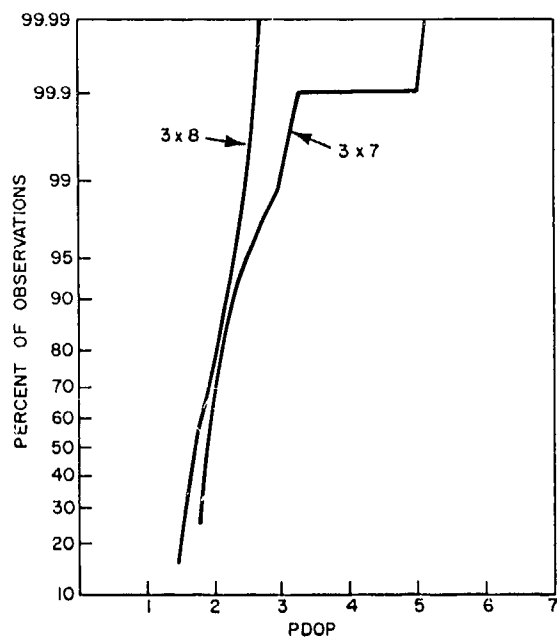


Fig. 40 — Percentile coverage versus PDOP value plot of  $3 \times 7$  and  $3 \times 8$  constellations at  $53^\circ$  inclination with a 24-hr period and a  $5^\circ$  mask

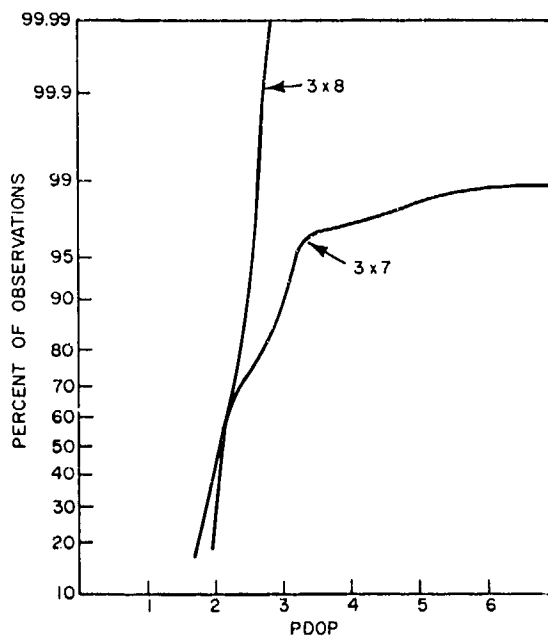


Fig. 41 — Percentile coverage versus PDOP value plot of  $3 \times 7$  and  $3 \times 8$  constellations at  $53^\circ$  inclination with a 24-hr period and a  $10^\circ$  mask

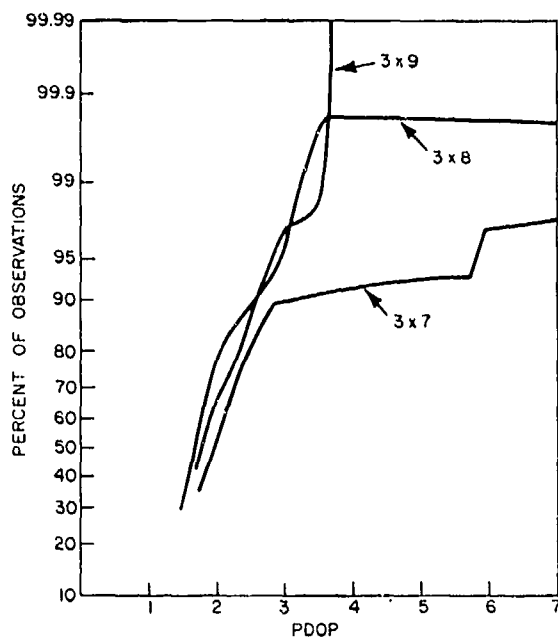


Fig. 42 — Percentile coverage versus PDOP value plot of  $3 \times 7$ ,  $3 \times 8$ , and  $3 \times 9$  constellations at  $90^\circ$  inclination with an 8-hr period and a  $5^\circ$  mask

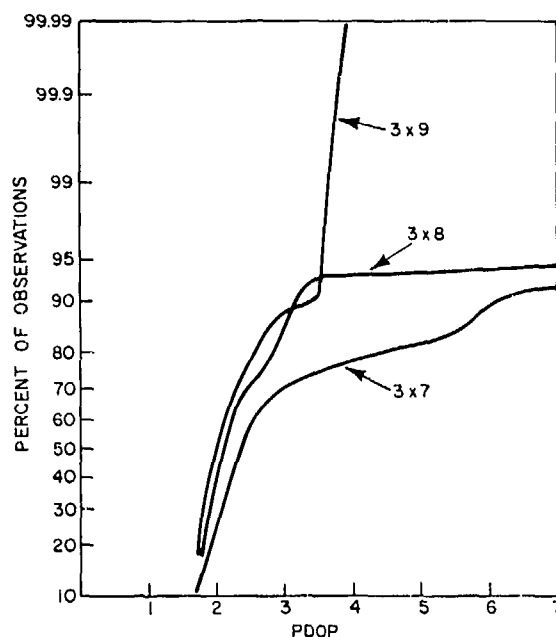


Fig. 43 — Percentile coverage versus PDOP value plot of  $3 \times 7$ ,  $3 \times 8$ , and  $3 \times 9$  constellations at  $90^\circ$  inclination with an 8-hr period and a  $10^\circ$  mask

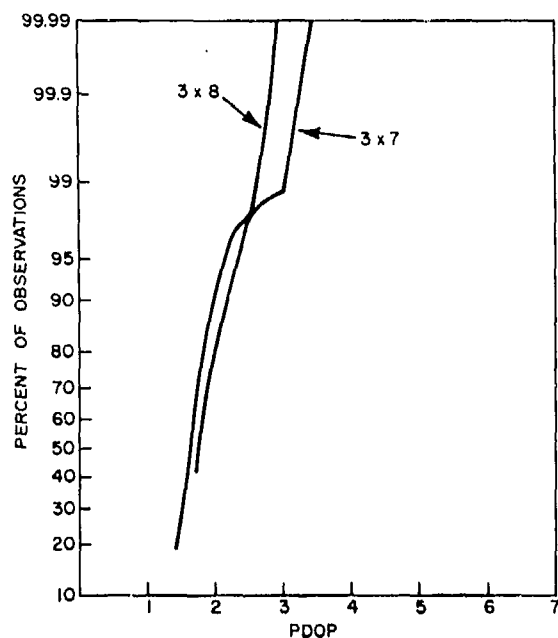


Fig. 44 — Percentile coverage versus PDOP value plot of  $3 \times 7$  and  $3 \times 8$  constellations at  $90^\circ$  inclination with a 24-hr period and a  $5^\circ$  mask

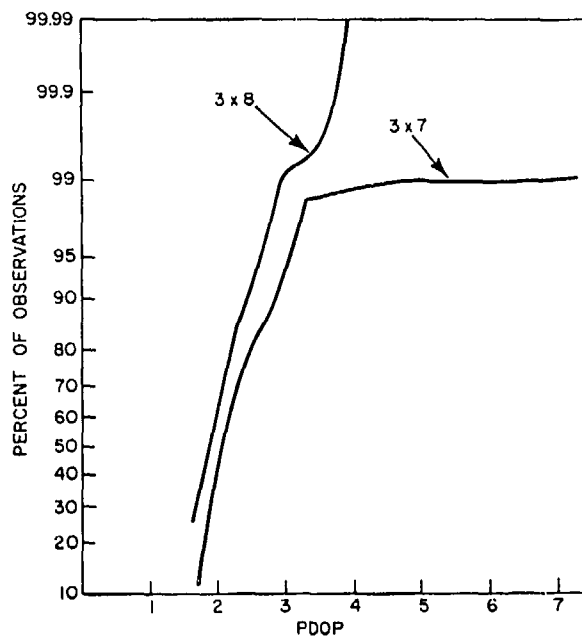


Fig. 45 — Percentile coverage versus PDOP value plot of  $3 \times 7$  and  $3 \times 8$  constellations at  $90^\circ$  inclination with a 24-hr period and a  $10^\circ$  mask

## CONCLUSIONS

1. GDOP results can be used to identify desirable constellation configurations and eliminate other configurations. However, other factors were considered in selecting a suitable constellation. These factors were satellite launch vehicle payload and booster requirements, ground station location, satellite size and power requirements, satellite clock stability, vulnerability to enemy attack, and overall operation.

2. A  $3 \times 9$  constellation with an 8- or 12-hour period and  $53^\circ$  inclination will give instantaneous, worldwide navigation capability 100% of the time for a  $5^\circ$  user elevation mask. This  $3 \times 9$  constellation will also withstand increases in the user elevation mask, and for any one satellite in the system which is inoperative no serious degradation of PDOP coverage will occur. This  $3 \times 9$  constellation is recommended as the prime candidate for use with the Timation Navigation Satellite Technique.

3. The worldwide PDOP coverage is a function of constellation inclination, and the constellation inclination may be chosen to minimize the PDOP values obtained. For a  $3 \times 9$  constellation with an 8-hr period this minimum value of PDOP occurs for an inclination of  $53^\circ$ . The PDOP dependence on inclination is reduced for a 24-hour orbit.

4. A  $3 \times 7$  constellation with an 8- to 24-hour period and an inclination from  $50^\circ$  to  $90^\circ$  will provide at least four satellites in view on a worldwide, continuous basis. The instantaneous three-dimensional navigational capability will be limited by high PDOP values about 10% of the time. The  $3 \times 7$  constellation PDOP coverage deteriorates with an increase in user elevation mask or with an inoperative satellite.

5. A  $3 \times 8$  constellation with an 8- to 24-hour period and a  $50^\circ$  to  $90^\circ$  inclination will provide better coverage than the corresponding  $3 \times 7$  constellation. The  $3 \times 8$  constellation is not the recommended constellation because increases in the user elevation mask or any one satellite in the inoperative causes more degradation of PDOP coverage than deemed desirable.

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<p>The Timation (<u>Time Navigation</u>) technique of passive ranging is a concept that could be expanded to provide a worldwide navigation service. Passive ranging is accomplished by measuring the time difference between electronic clocks located within the satellite and in the navigator's receiver. The Timation technique uses a highly stable, synchronized clock in the satellite; however, the navigator's clock stability may vary considerably, depending on the navigator's equipment and vehicle dynamics.</p> <p>One-hundred and seven satellite constellations are discussed in terms of their influence on earth coverage and navigation fix accuracy. The recommended configuration derived is comprised of three planes with nine satellites (3 x 9) each in 8-hour circular orbits at a 53° inclination.</p>			



Security Classification

14. KEY WORDS	LINK A		LINK B		LINK C	
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Timation Satellite navigation Constellation Passive range navigation Synchronized clocks Instantaneous position determination GDOP (geometric dilution of precision) Elevation mask Inoperative satellite constellation						

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